A STUDY ON METHODS TO APPLY THE ENERGY CONSERVATION THEOREM IN THE INVESTIGATION OF TRAFFIC ACCIDENTS USING DETAILED BREAKDOWN OF ENERGIES

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Abstract: *The road accidents are still part of our daily lives in an unwanted way and, in order to resolve the various resulting disputes, they need to be investigated in detail. If the investigation methods based on testimonials or/and visual observation are accessible even to amateurs, they also reveal a lot of subjectivity. The most reliable, professional and "cold" method of analyzing a traffic accident remains the scientific one based on the mathematical apparatus and studies specific to the field of accidentology science. We are going to present a possibility to apply the energy conservation theorem in the case of an road accident where we will attempt to thoroughly detail all the energies resulted from the impact.*

Keywords: *traffic accident, impact energy, detailed breakdown*

1. INTRODUCTION

Regardless of the progress made by current technology and driver education, with direct positive implications on the human-vehicle-environment system, road accidents will remain a major problem as long as hazard, technical imperfections and human lability are not completely eliminated. Even after the complete automation of transportation, if this ever happens, there will certainly still be traffic incidents because no program or technical achievement is perfect.

Consequently, accidentology, as a science, will never disappear and, moreover, it will have to keep up with the evolution of technology. And specialists will need to continuously improve themselves in the art of scientific investigation of traffic events. The intelligent accident analysis systems/programs have already been created (such as PC-Crash and others), but even these, regardless of how efficient they are, depend on the algorithms initially introduced, as well as on the concrete data relating to each individual event. Thus, specialists will have to perfect new and new scientific methods of technical analysis of accidents because these are the safest, the most professional and "cold" (devoid of feelings and subjectivity) methods of analyzing an accident because they are based on the standard mathematical-physical apparatus, as well as studies specific to the field of accidentology

Of course, traditional methods, based on observation, such as verifying the correspondence of damages and their complementarities according to the forensic principle of the shape, or analysis of the situation according to testimonial statements continues to play an important role, especially in the legal field, that is why the investigator is forced to consider them in parallel with the technical-scientific one, which is his main working tool.

In the technical-scientific investigations of the technical expertise resulted from accidents with impacts between two technical systems, the most used methods are the theorem of conservation of energies and the momentum conservation theorem. In this paper, in the case study presented as an example, a detailed developing of the energy method will be used, applying in this sense the energy conservation theorem in the new vision of thermodynamics according to [1]. Thus, it results that the total energy possessed by both cars before the collision (kinetic energies) should be found in the total energy output from the impact in the forms of kinetic energies, deformation energies, fracture energies, bending moments, shearing work, rotation/pivoting moments as a result of the eccentric impact, frictions, rotational kinetics, slewing etc.

2. CASE STUDY – PART ONE - ANALYSIS OF THE TRAFFIC ACCIDENT THROUGH THE METHODS OF TESTIMONIAL STATEMENTS AND VERIFICATION OF THE CORRESPONDENCE OF THE DAMAGES AND THEIR COMPLEMENTARITY ACCORDING TO THE FORENSIC PRINCIPLE OF THE SHAPE

The tactical situation to be analyzed (resulting from the official documents studied, the statements of those involved and the expert's on-site reconnaissance):

- one-way road, part of a boulevard with a central pedestrian area without slopes, in alignment, traffic flows on two lanes, lane 2 being shared with the tramway, lane 1 partially blocked by the rear of the bodies of larger vehicles legally parked obliquely at an angle of approximately 30-45 degrees to the longitudinal axis of the road near the right sidewalk;
- visible longitudinal road marking with a dashed line between the traffic lanes and specific parking marking obliquely on the right side of the street near sidewalk;
- asphalt road in very good condition, without bumps, possibly a longitudinal stress area for drivers on the lane 2 due to the presence of the tramway which makes many drivers prefer a driving position, illegal but considered more secure, between the traffic lanes in order to avoid some possible conflicts with the common carriageway with the tramway (to the left) and with the cars that are trying to leave the parking lot (to the right);
- there is a pedestrian sidewalk on both sides of the road;
- curbstones delimitation of the roadway plus oblique parking place;
- traffic regulated by signs;
- according to the statements, two cars arrived at the point of tension simultaneously and crossed paths;
- the two cars were a BMW 645Ci, it was traveling in the normal way of travel (Fig. 1-b) and the Alfa Romeo was intending to go back in reverse of the oblique parking lot without being guided (Fig. 1-a).

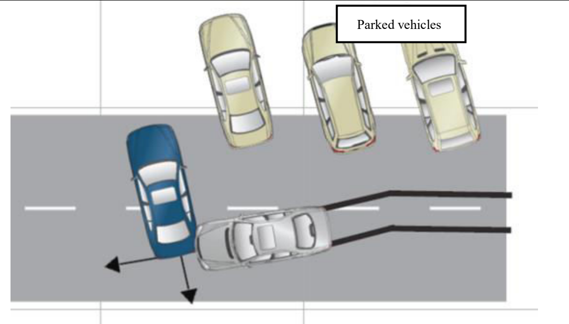
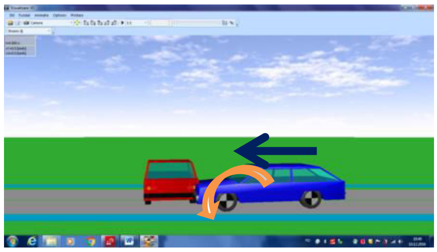
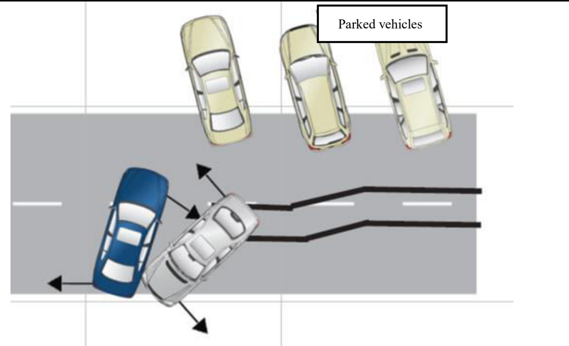
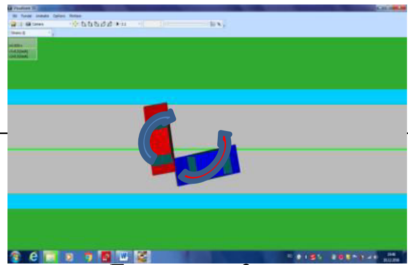
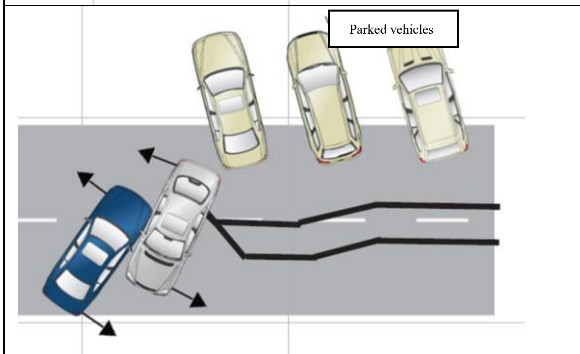
Based on the testimonial statements and the investigation of the damages suffered by both cars involved, the general scheme and evolution of the accident presented in Table 1 resulted [7, 10].

(a)

(b)

FIG. 1 Accident area – *Alfa Romeo/AR* driver's perspective (a) and *BMW* driver's perspective; (b) *Google Earth Pro* processing -

Table 1. The evolution of cars during the event

	Event stage 1 – The maximum engagement 
	Event stage 2 – The reaction of the cars following the primary impact, the separation after the first impact, the impact of the BMW with the curbstone and the trajectory followed towards the second impact between the cars 
	Event stage 3 – The second impact between the cars and the final position

3. CASE STUDY – PART TWO - TRAFFIC ACCIDENT ANALYSIS USING THE ENERGY METHOD (PHYSICAL MATHEMATICAL CALCULATION OF THE POSSIBILITY OF DAMAGES TO THE TWO CARS USING THE ENERGY CONSERVATION METHOD)

Applying the energy conservation theorem, the new vision according to [1], it results that the total energy possessed by both cars before the collision (kinetic energies) should be found in the total energy output from the impact in the forms of kinetic energies, deformation energies, fracture energies, bending moments, shearing work, rotation/pivoting moments as a result of the eccentric impact, frictions, rotational kinetics, slewing etc.

In other words:

$$\sum E_{input} = \sum E_{output} \quad (1)$$

It can be written as follows:

$$\sum E_{input} = E_{kineticBMW} + E_{kineticAR} \quad (2)$$

and also:

$$\sum E_{output} = \sum [E_{damages\ production} + (E_{pivoting} + E_{heat} + E_{noise})] \tag{2}$$

where, it must be taken into account that the damage to the BMW's bumper/front reinforcement consumed most of the energy.

A. Impact speed calculation for Alfa Romeo car

The distance traveled by the Alfa Romeo car from the moment of starting to the moment of impact - *d* is *4.2 meters*. The maximum possible acceleration (*a*) achievable by the type of vehicle in question, Alfa Romeo, is adopted to be *2.5 m/s²*. It follows that the maximum possible speed that the Alfa Romeo car could have had before the impact could have been:

$$v_{impact\ AR} = \sqrt{v_0^2 + 2ad} = \sqrt{0 + 2 * 2{,}5 * 4{,}2} \approx 4{,}5 m/sec \approx 16 km/h \tag{3}$$

It follows that the Alfa Romeo car could have collided with a maximum speed of *16 km/h.*

Taking into account the way to approach such maneuvers (see [2]), a speed of *8 km/h (2.22 m/sec)* will be adopted as the impact speed of the Alfa Romeo car.

B. Calculation of the energy required to deform the BMW front bumper reinforcement

The front bumper reinforcement, or, in other words, the bumper/protective bar, includes a supporting structure and a curved beam reinforcement (Fig. 3.1).

The total deformation energy of the bumper bar is composed of the energy required for bending under the $F_î$ effect (see Fig. 2) and the energy required for fracturing the welding bridges.

Within the bumper bar there are two types of welding seams:

- points-welding;
- fillet welding by material addition.

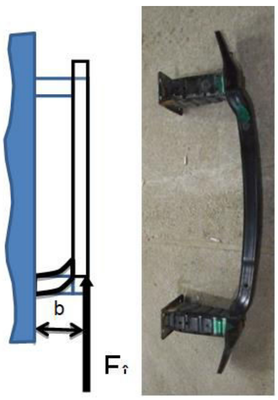


FIG. 2 BMW front reinforcement deformation energy calculation diagram

According to Fig. 2.6, the deformation under the statically applied force $F_î$ is:

$$\sigma_{st} = \frac{F_î * b}{E * A} \rightarrow F_î = \frac{\sigma_{st} * G * A}{b} = \frac{0{,}17 * 8{,}1 * 10^5 * 1 * 10^{-3}}{0{,}51} = 270 * 10^3\ N \tag{4}$$

$$E_{initial\ deformation} = M_î = F_î * b = 270 * 10^3 * 0{,}51 = 137{,}7 * 10^3\ Nm = 137{,}7 * 10^3 J \tag{5}$$

where:
- F_i – the force that produced the bending;
- b – brațul forței;
- A – the area of the bent section; it is approximated by the bending of the length of the sheet metal section from which the shock-absorbing (fixing) element of the reinforcement is formed; $A = l * g = 0{,}41 * 0{,}003 = 0{,}001m^2$
- G – modulus of elasticity in shear for carbon-steel.

Because there are two reinforcements that support the bumper and usually only 80% of the energy is absorbed:

$$E_{deformation} = 0{,}8 * \frac{E_{initial\ deformation}}{2} = 55{,}08 * 10^3 J \tag{6}$$

Calculation of the energy required to fracture weld bridges for additional welding areas, fillet welding:

The formula for calculating the load supported by the seam welding is:

$$P = 2 * 0{,}7 * s * l * \sigma_{as} \tag{7}$$

where:
- P – the load that can be supported by the welding seam (in our case the value of the minimum energy required to fracture the welding seam);
- s – the thickness of the plate to be welded;
- l – length of the broken welding seam (the approximate length was identified on the photographs);
- σ_{as} – strength of welding load.

So:

$$E_{fracture1} = P = 2 * 0{,}7 * 0{,}004 * 0{,}26 * 1050 * 10^4 = 15288J \tag{8}$$

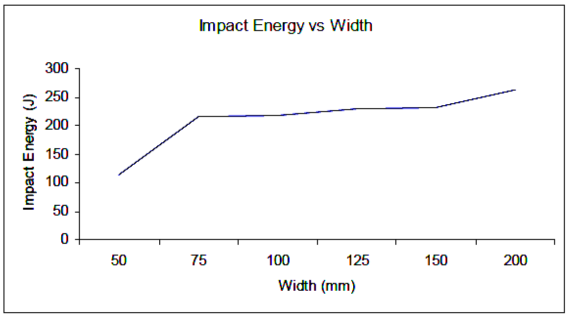


FIG. 3. The energy required to fracture point-welding bridges

Calculation of the energy required to fracture welding bridges for the points-welding areas will be made according to the tests presented in [3], the energy required to fracture point-welding bridges depending on the welding diameter is as shown in Fig. 3.

For a point-welding diameter of 5 mm, it results that an energy of approximately 120 J is required to fracture it. If we consider a number of 6 fractured welding points, the energy consumed for this is:

$$E_{fracture2} = 6 * 135 = 810J \tag{9}$$

The total energy for fracturing and deforming the bumper bar (front bumper reinforcement) is:

$$E_{bumper\ bar\ deformation} = E_{deformation} + E_{fracture1} + E_{fracture2} = \mathbf{71178\,J \approx 71kJ} \quad (10)$$

C. Calculation of the total energy as a result of the damages suffered by both cars

By extrapolating the calculation of the energy required to deform the BMW bumper onto the other damages to both cars, in which the most important are the deformations suffered by the Alfa Romeo car, mainly the deformation of the chassis, it is adopted:

$$E_{bumper\ bar\ deformation} = E_{deformation} + E_{fracture1} + E_{fracture2} = \mathbf{71178\,J \approx 71kJ} \quad (11)$$

$$\sum E_{output} = \sum [E_{damages\ production} + (E_{pivoting} + E_{heat} + E_{noise})] = E_{bumper\ bar\ deformation\ BMW} + E_{other\ damages\ AR+BMW} + E_{pivoting\ BMW+AR} + E_{heat\ BMW+AR} + E_{noise} = 71kJ + 90kJ = 161kJ \quad (12)$$

D. Calculating the impact speed for BMW car

The calculation of the impact speed for BMW car will be done using the deformation model used in the CRASH3 program [11]. Thus:

$$E_{col} = \frac{1}{2} m v_{BMW}^2 (1-\varepsilon^2) = E_A(1-\varepsilon^2) \quad (13)$$

where:

- E_{col} – the energy consumed in the collision;
- v_{BMW} - impact speed for BMW car;
- ε – collision coefficient;
- E_A – the kinetic energy of the BMW car held before impact.

The energy consumed in a collision is the sum of the energy of the deformation of the bumper and the kinetic energy of rotation/pivoting of the car as a result of the impact:

$$E_{col} = E_{bumper\ bar\ deformation\ BMW} + E_{kinetic\ of\ rotation\ BMW} + E_{other\ damages\ BMW} + E_{pivoting\ BMW} + E_{heat\ BMW} + E_{noise\ BMW} = \mathbf{79kJ + 5kJ = 84kJ} \quad (14)$$

The rotational kinetic energy of the car as a result of the impact is:

$$E_{kinetic\ of\ rotation} = \frac{m_{BMW} * v_{BMW}^2 * r^2}{2} = \frac{1895 * 7^2 * 4{,}5^2}{2} = 982J \quad (15)$$

Thus, the speed at impact of the BMW can be calculated:

$$v_{impactBMW} = \sqrt{\frac{2E_{col}}{m(1-\varepsilon^2)}} = \sqrt{\frac{2*(84kJ)}{1895kg(1-0{,}8^2)}} = \mathbf{15{,}69\ m/sec \approx 56{,}49km/h} \quad (16)$$

where:

- *ε = 0.8* is adopted because the first impact is considered more elastic due to the special construction of the bumper bar of the BMW car;
- BMW mass = 1815 kg (according to technical data) + 80 kg (driver's mass) = 1895 kg.

E. Determining the moment of time and the place of possible perception of the danger of an accident by the driver of the BMW car

Taking into account the largeness of the deformations of both cars involved, it is unlikely that the driver of the Alfa Romeo car actually took any evasive action before the impact.

The moment of perception of the dangerous situation by the BMW driver, due to the narrow width of the street (approximately 5 m) is considered to be the moment when he was certain that the Alfa Romeo is about to cross his direction of travel.

From that moment on, the Alfa Romeo car will travel a distance of approximately 4.2 m, with a uniformly accelerated speed that is considered to have reached a value of 8 km/h at the moment of impact. It will be considered uniformly accelerated rectilinear motion. Thus, the time (until impact) necessary to pass the *4.2 m* at a speed of *8 km/h* is:

$$t_{impact\ AR} = \frac{v_{impact\ AR}}{a_{AR}} = \frac{2,22}{2,5} = 0,88\ sec \tag{17}$$

The beginning of the avoidance action by the BMW driver occurs after the space traveled during his reaction time, $t_{reaction}$, which, for such situations, has been statistically shown to be *1-1.5 sec*. It follows that the BMW car driver had no time left to brake before the impact.

Thus, the conclusion is that the effective braking of the BMW driver occurred after the impact, simultaneously with the steering action to avoid it. Taking into account the calculation of the impact speed of the BMW car (*56.49 km/h*), as well as the inherent losses in the speed of moving in the longitudinal direction of the road as a result of the sudden evasive turn to the left, the speed of the BMW car when the danger was detected is estimated to be *60-65 km/h*.

Returning to the beginning of the demonstration…

The total energy that the two cars had when they collided is the sum of the kinetic energies of both cars.

$$\sum E_{input} = E_{kinetic\ BMW} + E_{kinetic\ AR} = \frac{m_{BMW} * v_{BMW}^2}{2} + \frac{m_{AR} * v_{AR}^2}{2} = \frac{1895 * 15,69^2}{2} + \frac{1280 * 2,22^2}{2} = 233,25kJ + 3,15kJ \approx 236,4kJ \tag{18}$$

As it can be observed:

$$\sum \boldsymbol{E}_{output} \approx \sum \boldsymbol{E}_{input} \tag{19}$$

because the difference $\mathbf{236,4kJ - 161kJ = 75,4kJ}$ is explainable by the terms that were not taken into account, namely: $E_{body\ shape\ deformation}$, $E_{rotational\ kinetic\ AR}$, $E_{other\ damages\ AR}$, $E_{pivoting\ AR}$, $E_{heat\ AR}$, $E_{noise\ AR}$, $E_{avoid\ steering\ BMW}$, $E_{braking\ BMW}$, $E_{tire\ lateral\ friction\ (AR+BMW)}$, $E_{lost/neglected\ at\ secondary\ impact}$ and $E_{calculation\ errors,\ tolerances,\ approximations}$.

The resulting conclusion after running the calculations: the energy balance confirms that the impact could have occurred under the conditions described in the testimonial statements and official documents.

CONCLUSIONS

As it can be easily seen, this analysis of a traffic accident case included two clearly delimited parts: the first one which was presented very briefly in point 2 of the paper is based on testimonial statements and other documents from the judicial file, as well as on the observation and in-depth analysis of the deformations of the vehicles (their bodyworks, wheels etc.) involved in the event. The other, presented in point 3 of the paper, aims to verify the scenario presented in point 2 and it is based exclusively on a physical-mathematical apparatus specific to the field of accidentology. More precisely, the principle of conservation of energy was used, the collision between two vehicles being more complex than a simple elastoplastic impact between two material bodies.

The originality of the research consists in detailing breakdown of the forms of energy resulting from the impact because in terms of the input energies they were clear and they were of the type of kinetic energies of two bodies in motion. And it is sufficient to recall the example of calculating the energy required to damage the bumper bar of the BMW car, which was identified as consisting of the energy required to bend it and the energy required to break the welding bridges.

The resulting energies of the impact were extremely complex. On the one hand, some have been calculated, but a lot of other components, listed in detail at the end of point 3, have remained uncalculated, the space of the present article not being enough. On the other hand, within the official judicial technical expertise, because of the space and time crisis these energies, which in our case have not been calculated, are statistically approximated based on the results of crash tests.

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